NAME _____ DATE _____ PERIOD _____

6-6

Study Guide

Modeling Real-World Data with Sinusoidal Functions

Example **The table shows the average monthly temperatures for Ann Arbor, Michigan. Write a sinusoidal function that models the average monthly temperatures, using $t = 1$ to represent January. Temperatures are in degrees Fahrenheit (°F).**

Jan.	30°
Feb.	34°
Mar.	45°
Apr.	59°
May	71°
June	80°
July	84°
Aug.	81°
Sept.	74°
Oct.	62°
Nov.	48°
Dec.	35°

These data can be modeled by a function of the form $y = A \sin (kt + c) + h$, where t is the time in months.

First, find A, h, and k.

A: $A = \frac{84 - 30}{2}$ or 27 — *A is half the difference between the greatest temperature and the least temperature.*

h: $h = \frac{84 + 30}{2}$ or 57 — *h is half the sum of the greatest value and the least value.*

k: $\frac{2\pi}{k} = 12$ — *The period is 12.*

$k = \frac{\pi}{6}$

Substitute these values into the general form of the function.

$y = A \sin (kt + c) + h$ $\qquad$ $y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$

To compute c, substitute one of the coordinate pairs into the equation.

$$y = 27 \sin \left(\frac{\pi}{6}t + c\right) + 57$$

$30 = 27 \sin \left[\frac{\pi}{6}(1) + c\right] + 57$ — *Use (t, y) = (1, 30).*

$-27 = 27 \sin \left(\frac{\pi}{6} + c\right)$ — *Subtract 57 from each side.*

$-\frac{27}{27} = \sin \left(\frac{\pi}{6} + c\right)$ — *Divide each side by 27.*

$\sin^{-1} (-1) = \frac{\pi}{6} + c$ — *Definition of inverse*

$\sin^{-1} (-1) - \frac{\pi}{6} = c$ — *Subtract $\frac{\pi}{6}$ from each side.*

$-2.094395102 \approx c$ — *Use a calculator.*

The function $y = 27 \sin \left(\frac{\pi}{6}t - 2.09\right) + 57$ is one model for the average monthly temperature in Ann Arbor, Michigan.

NAME _____ DATE _____ PERIOD _____

6-6

Practice

Modeling Real-World Data with Sinusoidal Functions

1. ***Meteorology*** The average monthly temperatures in degrees Fahrenheit (°F) for Baltimore, Maryland, are given below.

Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
32°	35°	44°	53°	63°	73°	77°	76°	69°	57°	47°	37°

a. Find the amplitude of a sinusoidal function that models the monthly temperatures.

b. Find the vertical shift of a sinusoidal function that models the monthly temperatures.

c. What is the period of a sinusoidal function that models the monthly temperatures?

d. Write a sinusoidal function that models the monthly temperatures, using $t = 1$ to represent January.

e. According to your model, what is the average temperature in July? How does this compare with the actual average?

f. According to your model, what is the average temperature in December? How does this compare with the actual average?

2. ***Boating*** A buoy, bobbing up and down in the water as waves move past it, moves from its highest point to its lowest point and back to its highest point every 10 seconds. The distance between its highest and lowest points is 3 feet.

a. What is the amplitude of a sinusoidal function that models the bobbing buoy?

b. What is the period of a sinusoidal function that models the bobbing buoy?

c. Write a sinusoidal function that models the bobbing buoy, using $t = 0$ at its highest point.

d. According to your model, what is the height of the buoy at $t = 2$ seconds?

e. According to your model, what is the height of the buoy at $t = 6$ seconds?