Lesson 7-1

Example 1

Prove that sec *x* cot *x* = sin *x* is *not* a trigonometric identity by producing a counterexample.

Suppose $x = \frac{\pi}{3}$.

$$\sec x \cot x \stackrel{?}{=} \sin x$$

$$\sec \frac{\pi}{3} \cot \frac{\pi}{3} \stackrel{?}{=} \sin \frac{\pi}{3} \qquad \textit{Replace } x \textit{ with } \frac{\pi}{3}.$$

$$\left(\frac{1}{\frac{1}{2}}\right)\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) \stackrel{?}{=} \frac{\sqrt{3}}{2}$$

$$(2)\left(\frac{\sqrt{3}}{3}\right) \stackrel{?}{=} \frac{\sqrt{3}}{2}$$

$$\frac{2\sqrt{3}}{3} \neq \frac{\sqrt{3}}{2}$$

Example 2

Use the given information to find the trigonometric value.

a. If cot $\theta = \frac{6}{5}$, find tan θ.

$$\tan \theta = \frac{1}{\cot \theta} \qquad \textit{Choose an identity that involves tan } \theta \textit{ and cot } \theta.$$

$$= \frac{1}{\frac{6}{5}} \text{ or } \frac{5}{6} \qquad \textit{Substitute } \frac{6}{5} \textit{ for cot } \theta \textit{ and evaluate.}$$

b. If sec $\theta = \frac{5}{4}$, find cot θ.

Since there are no identities relating sec θ and cot θ, we must use two identities, one relating sec θ and tan θ and another relating cot θ and tan θ.

$\sec^2 \theta = 1 + \tan^2 \theta$ *Pythagorean identity*

$\left(\frac{5}{4}\right)^2 = 1 + \tan^2 \theta$ *Substitute $\frac{5}{4}$ for sec θ.*

$\frac{25}{16} = 1 + \tan^2 \theta$

$\frac{9}{16} = \tan^2 \theta$

$\pm\frac{3}{4} = \tan \theta$

Now find cot θ.

$\cot \theta = \frac{1}{\tan \theta}$ *Reciprocal identity*

$\pm\frac{4}{3}$

Example 3

Express each value as a trigonometric function of an angle in Quadrant I.

a. sin 765°

765° and 45° differ by a multiple of 360°.

$765° = 45° + 2(360°)$

$\sin 765° = \sin (45° + 2(360°))$ *Case 1, with A = 45° and k = 2*

$= \sin 45°$

b. $\sin\left(-\frac{19\pi}{3}\right)$

The sum of $-\frac{19\pi}{3}$ and $\frac{\pi}{3}$ is a multiple of 2π.

$-\frac{19\pi}{3} = -6\pi - \frac{\pi}{3}$

$\sin\left(-\frac{19\pi}{3}\right) = \sin\left(-6\pi - \frac{\pi}{3}\right)$ *Case 3, with A = $\frac{\pi}{3}$ and k = -3*

$= -\sin\frac{\pi}{3}$

c. cos 935°

Relate 935°

$935° = 35° + 5(180°)$ *935° and 35° differ by an odd multiple of 180°.*

$\cos 935° = \cos (35° + 5(180°))$ *Case 2, with A = 35° and k = 3*

$= -\cos 35°$

d. cot $\frac{11\pi}{4}$

The sum of $\frac{11\pi}{4}$ and $\frac{\pi}{4}$, which is $\frac{12\pi}{4}$ or 3π, is an odd multiple of π.

$\frac{11\pi}{4} = 3\pi - \frac{\pi}{4}$ *Case 4 with A = $\frac{\pi}{4}$ and k = 2*

$\cot \frac{11\pi}{4} = \frac{\cos \frac{11\pi}{4}}{\sin \frac{11\pi}{4}}$ *Rewrite using the quotient identity.*

$= \frac{\cos 13\pi - \frac{\pi}{4}2}{\sin 13\pi - \frac{\pi}{4}2}$

$= \frac{-\cos \frac{\pi}{4}}{\sin \frac{\pi}{4}}$ or $-\cot \frac{\pi}{4}$ *Quotient identity*

Example 4

Simplify cos *x* cot *x* + sin *x*.

$\cos x \cot x - \sin x = \cos x \cdot \frac{\cos x}{\sin x} + \sin x$ *Definition of cot x*

$= \frac{\cos^2 x}{\sin x} + \sin x$

$= \frac{1 - \sin^2 x}{\sin x} + \sin x$ *Pythagorean identity: $\sin^2 x + \cos^2 x = 1$*

$= \frac{1}{\sin x} - \sin x + \sin x$

$= \frac{1}{\sin x}$ or $\csc x$ *Reciprocal identity*

Example 5

PHYSICS **When an object sits at rest, there is no force of friction working on the object. Once a force is applied to slide the object, frictional force is generated. The force of friction opposes the force being applied to the object, and is always equal to this applied force until the object begins to move. When the object begins to slide, applied force becomes greater than the maximum force of friction, F_f^{MAX}. The coefficient of static friction μ_s is the ratio of the maximal force of friction F_f^{MAX} to the normal force F_N, or $\mu_S = \dfrac{F_f^{MAX}}{F_N}$.**

a. Simplify the equation for coefficient of static friction if $F_f^{MAX} = mg \sin \theta$ and $F_N = mg \cos \theta$.

b. Suppose a block is sitting on a flat surface. The surface is raised at one end to form an angle $\theta = 60°$ with the ground at which point the block begins to slip down the surface. When this happens, the force of friction is at its maximum. What is the coefficient of static friction?

a. $\mu_S = \dfrac{F_f^{MAX}}{F_N}$

$\mu_S = \dfrac{mg \sin \theta}{mg \cos \theta}$

$\mu_S = \tan \theta$

b. $\mu_S = \tan 60°$

$\mu_S = \sqrt{3}$

$\mu_S \approx 1.732050808$

The coefficient of static friction is about 1.73.