Lesson 7-3

Example 1

a. Show by producing a counterexample that cos (x - y) ≠ cos x - cos y.

b. Show that the difference identity for cosine is true for the values used in part a.

a. Let $x = 0$ and $y = \frac{\pi}{4}$. First find cos (x - y) for $x = 0$ and $y = \frac{\pi}{4}$.

$\cos(x - y) = \cos 1 0 - \frac{\pi}{4} 2$ *Replace x with 0 and y with $\frac{\pi}{4}$.*

$= \cos 1 - \frac{\pi}{4} 2$ *$0 - \frac{\pi}{4} = -\frac{\pi}{4}$*

$= \frac{\sqrt{2}}{2}$

Now find cos x - cos y.

$\cos x - \cos y = \cos 0 - \cos \frac{\pi}{4}$ *Replace x with 0 and y with $\frac{\pi}{4}$.*

$= 1 - \frac{\sqrt{2}}{2}$ or $\frac{2 - \sqrt{2}}{2}$

So, cos (x - y) ≠ cos x - cos y.

b. Show that $\cos(x - y) = \cos x \cos y + \sin x \sin y$ for $x = 0$ and $y = \frac{\pi}{4}$.

First find cos (x - y). From part a, we know that $\cos 1 0 - \frac{\pi}{4} 2 = \frac{\sqrt{2}}{2}$.

Now find $\cos x \cos y + \sin x \sin y$.

$\cos x \cos y + \sin x \sin y = \cos 0 \cos \frac{\pi}{4} + \sin 0 \sin \frac{\pi}{4}$ *Substitute for x and y.*

$= (1) 1 \frac{\sqrt{2}}{2} 2 + (0) 1 \frac{\sqrt{2}}{2} 2$

$= \frac{\sqrt{2}}{2}$

Thus, the difference identity for cosine is true for $x = 0$ and $y = \frac{\pi}{4}$.

Example 2

Use the sum or difference identity for cosine to find the exact value of cos 825°.

$825° = 2(360°) + 105°$ *Symmetry identity, Case 1*

$\cos 825° = \cos 105°$

$\cos 105° = \cos(60° + 45°)$ *60° and 45° are two common angles whose sum is 105°.*

$= \cos 60° \cos 45° - \sin 60° \sin 45°$ *Sum identity for cosine*

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

Therefore, $\cos 825° = \frac{\sqrt{2} - \sqrt{6}}{4}$.

Example 3

Find the value of sin ($x + y$) if $0 < x < \frac{\pi}{2}$, $0 < y < \frac{\pi}{2}$, sin $x = \frac{12}{13}$ and sin $y = \frac{15}{17}$.

In order to use the sum identity for sine, we need to know cos x and cos y. We can use a Pythagorean identity to determine the necessary values.

$\sin^2 \alpha + \cos^2 \alpha = 1 \rightarrow \cos^2 \alpha = 1 - \sin^2 \alpha$ *Pythagorean identity*

Since we are given that the angles in Quadrant I, the values of sine and cosine are positive. Therefore, $\cos \alpha = \sqrt{1 - \sin^2 \alpha}$.

$\cos x = \sqrt{1 - \left(\frac{12}{13}\right)^2}$

$= \sqrt{\frac{25}{169}}$ or $\frac{5}{13}$

$\cos y = \sqrt{1 - \left(\frac{15}{17}\right)^2}$

$= \sqrt{\frac{64}{289}}$ or $\frac{8}{17}$

Now substitute these values in to the difference identity for sine.

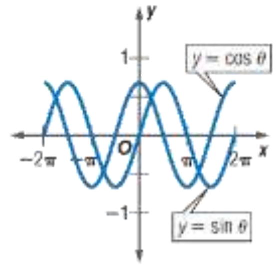
$\sin(x + y) = \sin x \cos y + \cos x \sin y$

$= 1\frac{12}{13}2 1\frac{8}{17}2 + 1\frac{5}{13}2 1\frac{15}{17}2$

$= \frac{171}{221}$ or about 0.774

Example 4

TRIGONOMETRY If the sine wave is shifted $\frac{\pi}{2}$ units to the left, it coincides with the cosine wave. Therefore, $\cos\theta = \sin 1\theta + \frac{\pi}{2}2$ is an identity. Prove the identity.



$\cos\theta = \sin 1\theta + \frac{\pi}{2}2$

$\cos\theta = \sin\theta\cos\frac{\pi}{2} + \cos\theta\sin\frac{\pi}{2}$

$\cos\theta = \sin\theta(0) + \cos\theta(1)$

$\cos\theta = \cos\theta$

The identity is true.

Example 5

Use the sum or difference identity for tangent to find the exact value of tan 345°.

tan 345°	= tan (300° + 45°)	*300° and 45° are two common angles whose sum is 345°.*
	$= \frac{\tan 300° + \tan 45°}{1 - \tan 300° \tan 45°}$	*Sum identity for tangent*
	$= \frac{-\sqrt{3} + 1}{[1 - (-\sqrt{3})](1)}$	*tan 300° = $-\sqrt{3}$, tan 45° = 1*
	$= -2 + \sqrt{3}$	*Multiply by $\frac{1 - \sqrt{3}}{1 - \sqrt{3}}$ to simplify.*

Example 6

Verify that $\cot\left(\frac{\pi}{4} - A\right) = \frac{\cos A + \sin A}{\cos A - \sin A}$.

Transform the left side since it is more complicated.

$$\cot\left(\frac{\pi}{4} - A\right) \stackrel{?}{=} \frac{\cos A + \sin A}{\cos A - \sin A}$$

$$\frac{1}{\tan\left(\frac{\pi}{4} - A\right)} \stackrel{?}{=} \frac{\cos A + \sin A}{\cos A - \sin A}$$ *Reciprocal identity*

$$\frac{1 + \tan\frac{\pi}{4}\tan A}{\tan\frac{\pi}{4} - \tan A} \stackrel{?}{=} \frac{\cos A + \sin A}{\cos A - \sin A}$$ *Difference identity for tangent*

$$\frac{1 + (1)\tan A}{1 - \tan A} \stackrel{?}{=} \frac{\cos A + \sin A}{\cos A - \sin A}$$ *$\tan\frac{\pi}{4} = 1$*

$$\frac{1 + \frac{\sin A}{\cos A}}{1 - \frac{\sin A}{\cos A}} \stackrel{?}{=} \frac{\cos A + \sin A}{\cos A - \sin A}$$ *$\tan A = \frac{\sin A}{\cos A}$*

$$\frac{\cos A + \sin A}{\cos A - \sin A} = \frac{\cos A + \sin A}{\cos A - \sin A}$$ *Multiply by cos A.*
