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Example 1

Solve $\sqrt{3}\cos x + 2\tan x \cos^2 x = 0$ for principal values of x. Express solutions in degrees.

$$\sqrt{3}\cos x + 2\tan x \cos^2 x = 0$$
$$\sqrt{3}\cos x + 2\frac{\sin x}{\cos x}\cos^2 x = 0 \qquad \tan x = \frac{\sin x}{\cos x}$$
$$\sqrt{3}\cos x + 2\sin x \cos x = 0$$
$$\cos x\,(\sqrt{3} + 2\sin x) = 0 \qquad \textit{Factor.}$$

$\cos x = 0$
$x = 90°$

$\sqrt{3} + 2\sin x = 0$
$\sin x = -\frac{\sqrt{3}}{2}$
$x = -60°$

Example 2

Solve $\tan x \sin^2 x + \tan x \cos^2 x = \tan^2 x$ for $0 \le \pi \le 2\pi$.

This equation can be written in terms of tan x only.

$$\tan x \sin^2 x + \tan x \cos^2 x = \tan^2 x$$
$$\tan x\,(\sin^2 x + \cos^2 x) = \tan^2 x \qquad \textit{Factor.}$$
$$\tan x\,(1) = \tan^2 x \qquad \textit{Pythagorean identity: } \sin^2 x + \cos^2 x = 1$$
$$\tan x - \tan^2 x = 0$$
$$\tan x\,(1 - \tan x) = 0 \qquad \textit{Factor.}$$

$\tan x = 0$
$x = 0$ or $x = \pi$

$1 - \tan x = 0$
$\tan x = 1$
$x = \frac{\pi}{4}$ or $x = \frac{5\pi}{4}$

The solutions are $0, \frac{\pi}{4}, \pi$, and $\frac{5\pi}{4}$.

Example 3

Solve $\cos^2 x \sin x - \frac{1}{2}\sin x + \cos^2 x - \frac{1}{2} = 0$ for all real values of x.

Use factoring to solve this equation.

$$\cos^2 x \sin x - \frac{1}{2}\sin x + \cos^2 x - \frac{1}{2} = 0$$

$$\sin x\, 1 \cos^2 x - \frac{1}{2}2 + \cos^2 x - \frac{1}{2} = 0 \qquad \textit{Factor.}$$

$$1 \cos^2 x - \frac{1}{2}2(\sin x + 1) = 0$$

$$\cos^2 x - \frac{1}{2} = 0$$

$$\cos^2 x = \frac{1}{2}$$

$$\cos x = \pm\frac{\sqrt{2}}{2}$$

$$x = \frac{\pi}{4} + \frac{\pi}{2}k$$

$$\sin x + 1 = 0$$

$$\sin x = -1$$

$$x = \frac{3\pi}{2} + 2\pi k$$

The solutions are $\frac{\pi}{4} + \frac{\pi}{2}k$ and $\frac{3\pi}{2} + 2\pi k$.

Example 4

If the range (horizontal distance traveled) R of a projectile is 10,000 m and the initial velocity v_0 is 500 m/s, find θ, the angle at which the projectile was launched, given that $g = 9.8$ m/s^2. The formula that relates the variables is $R = \frac{v_0^2 \sin 2\theta}{g}$.

$$R = \frac{v_0^2 \sin 2\theta}{g}$$

$$10{,}000 = \frac{(500)^2 \sin 2\theta}{9.8} \qquad \textit{Replace } v_0 \textit{ with 500 and } g \textit{ with 9.8.}$$

$$0.392 = \sin 2\theta$$

$$2\theta = 23.079$$

$$\theta \approx 11.54 + 360°k$$

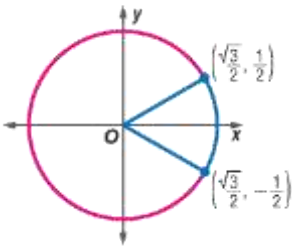
The least positive value for θ is obtained by letting $k = 0$. Therefore, $\theta \approx 11.54$.

Example 5

Solve $2\cos\theta - \sqrt{3} < 0$ for $0 \le \theta \le 2\pi$.

$$2\cos\theta - \sqrt{3} < 0$$
$$2\cos\theta < \sqrt{3}$$
$$\cos\theta < \frac{\sqrt{3}}{2}$$ *Solve for cos θ.*

In terms of the unit circle, we need to find points with x-coordinates less than $\frac{\sqrt{3}}{2}$.



The values of θ for which $\cos\theta < \frac{\sqrt{3}}{2}$ are $\frac{\pi}{6}$ and $\frac{11\pi}{6}$. The figure shows that the solution of the inequality is $\frac{\pi}{6} \le \theta \le \frac{11\pi}{6}$.