File LibraryLesson 7-7

Example 1

Find the distance between *P*(-1, 3) and the line with equation 3*x* - 4*y* = 10.

First rewrite the equation of the line in standard form.

$3x - 4y = 10 \longrightarrow 3x - 4y - 10 = 0$

Then use the formula for the distance from a point to a line.

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d = \frac{3(-1) + (-4)(3) - 10}{\pm\sqrt{3^2 + (-4)^2}} \qquad A = 3, B = -4, C = -10, x_1 = -1, y_1 = 3$$

$$d = \frac{-25}{-\sqrt{25}} \text{ or } -5 \qquad \text{Since } C \text{ is negative, use } \sqrt{A^2 + B^2}.$$

Therefore, *P* is 5 unit from the line 3*x* - 4*y* = 10. Since *d* is negative, *P* is on the same side of the line as the origin.

Example 2

Find the distance between the lines with equations 2*x* + 3*y* = 5 and $y = -\frac{2}{3}x + 2$.

Since $y = -\frac{2}{3}x + 2$ is in slope-intercept form, we know that it passes through the point at (0, 2). Use this point to find the distance to the other line.

The standard form of the other equation is 2*x* + 3*y* - 5 = 0.

$$d = \frac{Ax_1 + By_1 + C}{\pm\sqrt{A^2 + B^2}}$$

$$d = \frac{2(0) + 3(2) - 5}{\pm\sqrt{2^2 + 3^2}} \qquad A = 2, B = 3, C = -5, x_1 = 0, y_1 = 2$$

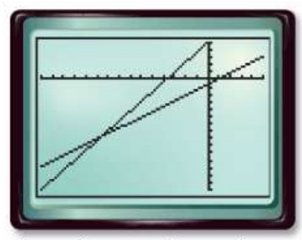
$$d = \frac{1}{\sqrt{13}} \text{ or about } 0.28 \qquad \text{Since } C \text{ is negative, use } \sqrt{A^2 + B^2}.$$

The distance between the lines is about 0.28 unit.

Example 3

Find equations of the lines that bisect the angles formed by the lines $4x - 3y = -15$ and $6x - 8y = 5$.

Graph each equation. Note the location of the origin. The origin is in the interior of the acute angle.



[15, 5] scl: 1 by [15, 5] scl: 1

Bisector of the acute angle

$d_1 = d_2$

$$d_1 = \frac{4x_1 - 3y_1 + 15}{-\sqrt{4^2 + (-3)^2}}$$

$$d_2 = \frac{6x_1 - 8y_1 - 5}{\sqrt{6^2 + (-8)^2}}$$

$$\frac{4x_1 - 3y_1 + 15}{-5} = \frac{6x_1 - 8y_1 - 5}{10}$$

Bisector of the obtuse angle

$d_1 = -d_2$

$$d_1 = \frac{4x_1 - 3y_1 + 15}{-\sqrt{4^2 + (-3)^2}}$$

$$d_2 = \frac{6x_1 - 8y_1 - 5}{\sqrt{6^2 + (-8)^2}}$$

$$\frac{4x_1 - 3y_1 + 15}{-5} = -\frac{6x_1 - 8y_1 - 5}{10}$$

Simplifying and dropping the subscripts yields $-14x + 14y = 25$ and $-2x - 2y = 35$, respectively.

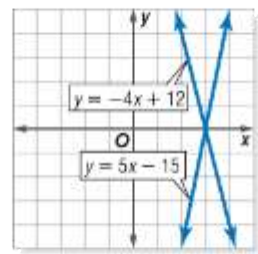
Example 4

OPTICS Refer to the application at the beginning of the Lesson 7-8 in your book. Suppose that the light emanates from point (5, 10) then strikes the mirror at (3, 0) and passes through the point at (5, -8).

a. Determine an equation of the line on which the mirror lies.

b. Use the equation to determine the angle the mirror makes with the *x*-axis.

a. Using the points at (5, 10), (5, -8), and (3, 0) and the slope-intercept form , you can find that the light travels along lines with equations $y = 5x - 15$ and $y = -4x + 12$. In standard form, these are $5x - y - 15 = 0$ and $4x + y - 12 = 0$.



The mirror lies on the angle bisector of the acute angles formed by these lines.

The origin is not contained in the angle we wish to bisect, so we use $d_1 = -d_2$.

$$\frac{5x - y - 15}{\sqrt{26}} = -\frac{4x + y - 12}{\sqrt{17}}$$

$$\sqrt{17}(5x - y - 15) = -\sqrt{26}(4x + y - 12)$$

$$(5\sqrt{17} + 4\sqrt{26})x + (-\sqrt{17} + \sqrt{26})y - 15\sqrt{17} - 12\sqrt{26} = 0$$

The mirror lies on the line with equation $(5\sqrt{17} + 4\sqrt{26})x + (-\sqrt{17} + \sqrt{26})y - 15\sqrt{17} - 12\sqrt{26} = 0$

b. The slope of the line on which the mirror lies is $-\dfrac{5\sqrt{17} + 4\sqrt{26}}{-\sqrt{17} + \sqrt{26}}$ or $-21 - \sqrt{442}$.

Recall that the slope of a line is the tangent of the angle the line makes with the *x*-axis. Since $\tan\theta = -21 - \sqrt{442}$, the positive value for θ is approximately 91.4°.

The mirror makes an angle of 91.4° with the positive *x*-axis.