Lesson 8-7 Modeling Motions Using Parametric Equations

Example 1

Find the initial horizontal velocity and vertical velocity of a ball kicked with an initial velocity of 15 feet per second at an angle of 40°.

$|\overrightarrow{\mathbf{v}_x}| = |\vec{\mathbf{v}}| \cos \theta$
$|\overrightarrow{\mathbf{v}_x}| = 15 \cos 40°$
$|\overrightarrow{\mathbf{v}_x}| \approx 11.5$

$|\overrightarrow{\mathbf{v}_y}| = |\vec{\mathbf{v}}| \sin \theta$
$|\overrightarrow{\mathbf{v}_y}| = 15 \sin 40°$
$|\overrightarrow{\mathbf{v}_y}| \approx 9.6$

The initial horizontal velocity is about 11.5 feet per second and the initial vertical velocity is about 9.6 feet per second.

Example 2

SPORTS **Suppose a professional soccer player kicks a ball with an initial velocity of 31 yards per second at an angle of 60° to the horizontal. Suppose a player on the opposing team comes into contact with the ball 5 seconds later. How far has the ball traveled horizontally and what is its vertical height at that time?**

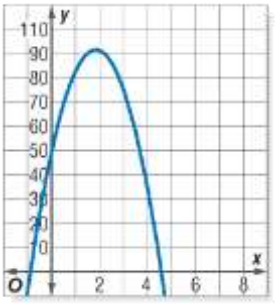
Write the position of the ball as a pair of parametric equations defining the path of the ball for any time t in seconds.

$x = t|\vec{\mathbf{v}}| \cos \theta$ $\quad$ $y = t|\vec{\mathbf{v}}| \sin \theta - \frac{1}{2}gt^2$

The initial velocity of 31 yards per second must be expressed as 93 feet per second as gravity is expressed in terms of feet per second squared.

$x = t(93) \cos 60°$
$x = 93t \cos 60°$

$y = t(93) \sin 60° - \frac{1}{2}(32)t^2$
$y = 93t \sin 60° - 16t^2$



Find x and y when $t = 5$.

$x = 93(5) \cos 60°$
$x \approx 232.5$

$y = 93(5) \sin 60° - 16(5^2)$
$y \approx 2.7$

After 5 seconds, the soccer ball has traveled 232.5 feet or 77.5 yards horizontally and is about 2.7 feet or 0.9 yard above the ground.

Example 3

PITCHING **The pitcher in an adult baseball league throws the ball at an angle of 5° with the horizontal at a speed of 55 miles per hour. The distance from the pitcher's mound to home plate is 45 feet. If the pitcher releases the ball 2.9 feet above the ground, how far above the ground is the ball when it crosses home plate?**

First, write parametric equations that model the path of the baseball. Remember to convert 55 mph to about 80.7 feet per second.

$x = t|\vec{\mathbf{v}}| \cos\theta$ $\qquad$ $y = t|\vec{\mathbf{v}}| \sin\theta - \frac{1}{2}gt^2 + h$

$x = t(80.7) \cos 5°$ $\qquad$ $y = t(80.7) \sin 5° - \frac{1}{2}(32)t^2 + 2.9$

$x = 80.7t \cos 5°$ $\qquad$ $y = 80.7t \sin 5° - 16t^2 + 2.9$

Then, find the amount of time that it will take the baseball to travel 45 feet horizontally. This will be the moment when it crosses home plate.

$$45 = 80.7t \cos 5°$$
$$t = \frac{45}{80.7 \cos 5°}$$
$$t \approx 0.565$$

The baseball will cross home plate in about 0.56 seconds.

To find the vertical position of the ball at that time, find y when $t = 0.56$.

$y = 80.7t \sin 5° - 16t^2 + 2.9$
$y = 80.7(0.56) \sin 5° - 16(0.56)^2 + 2.9$
$y \approx 1.821$

The baseball will be about 1.8 feet above home plate.