

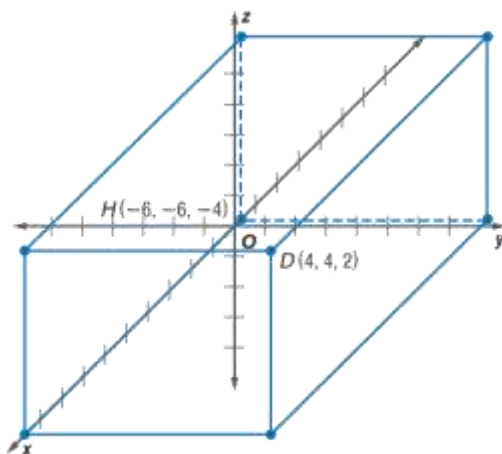
## Lesson 8-8 Transformational Matrices in Three-Dimensional Space

**Example 1**

The vertices of a rectangular prism are given by  $A(4, 4, -4)$ ,  $B(4, -4, -4)$ ,  $C(4, -4, 2)$ ,  $D(4, 4, 2)$ ,  $E(-6, -4, 2)$ ,  $F(-6, 4, 2)$ ,  $G(-6, 4, -4)$ , and  $H(-6, -4, -4)$ . Represent these vertices in a vertex matrix. Then graph the rectangular prism.

The vertex matrix for the prism is

	$A$	$B$	$C$	$D$	$E$	$F$	$G$	$H$
$x$	4	4	4	4	-6	-6	-6	-6
$y$	4	-4	-4	4	-4	4	4	-4
$z$	-4	-4	2	2	2	2	-4	-4

$$\begin{bmatrix} 4 & 4 & 4 & 4 & -6 & -6 & -6 & -6 \\ 4 & -4 & -4 & 4 & -4 & 4 & 4 & -4 \\ -4 & -4 & 2 & 2 & 2 & 2 & -4 & -4 \end{bmatrix}.$$


**Example 2**

Bob needs to translate a prism using the vector  $\vec{a} = \langle 2, 2, 0 \rangle$ . The vertices of the prism have the following coordinates.

$$A(3, 2, -2) \quad B(0, 0, -3) \quad C(-1, 4, -3)$$

$$D(-1, 4, 4) \quad E(3, 2, 4) \quad F(0, 0, 4)$$

- a. Write a matrix that will have such an effect on the figure.
  - b. Find the coordinates of the vertices of the translated image.
- a. To translate the prism by the vector  $\vec{a} = \langle 2, 2, 0 \rangle$ , we must first add 2 to each of the  $x$ - and  $y$ -coordinates. The  $z$ -coordinates remain the same. The translation matrix can be written as
- $$\begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$
- b. Write the vertices of the prism in a  $6 \times 3$  matrix. Then add it to the translation matrix to find the vertices of the translated image.

$$\begin{array}{c} \text{Original Matrix} \\ \begin{bmatrix} 3 & 0 & -1 & -1 & 3 & 0 \\ 2 & 0 & 4 & 4 & 2 & 0 \\ -2 & -3 & -3 & 4 & 4 & 4 \end{bmatrix} \end{array} + \begin{array}{c} \text{Translation Matrix} \\ \begin{bmatrix} 2 & 2 & 2 & 2 & 2 & 2 \\ 2 & 2 & 2 & 2 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \text{Translated Image Matrix} \\ \begin{bmatrix} 5 & 2 & 1 & 1 & 5 & 2 \\ 4 & 2 & 6 & 6 & 4 & 2 \\ -2 & -3 & -3 & 4 & 4 & 4 \end{bmatrix} \end{array}$$

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**Example 3**

Let  $M$  represent the vertex matrix of the rectangular prism in Example 1.

- a. Find  $TM$  if  $T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ .
- b. Graph the resulting image.

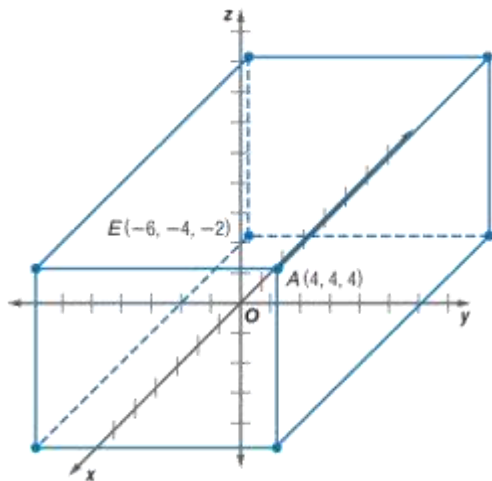
- c. Describe the transformation represented by matrix  $T$ .

- a. First find  $TM$ .

$$TM = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 4 & 4 & 4 & 4 & -6 & -6 & -6 & -6 \\ 4 & -4 & -4 & 4 & -4 & 4 & 4 & -4 \\ -4 & -4 & 2 & 2 & 2 & 2 & -4 & -4 \end{bmatrix}$$

$$TM = \begin{bmatrix} 4 & 4 & 4 & 4 & -6 & -6 & -6 & -6 \\ 4 & -4 & -4 & 4 & -4 & 4 & 4 & -4 \\ 4 & 4 & -2 & -2 & -2 & -2 & 4 & 4 \end{bmatrix}$$

- b. Then graph the points represented by the resulting matrix.



- c. The transformation matrix  $T$  reflects the image of each vertex over the  $xy$ -plane. This results in a reflection of the prism when the new vertices are connected by segments.